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M.Sc 4th Semester Examination, 2020

Applied Mathematicswith Oceanology and Computer Programming

(Functional Analysis)

Paper: MTM – 401

Full Marks: 50

Time: 2 hours

Answer **Q. No. 1** and any **four**questions from the rest.

The figures in the right hand margin indicate marks

1. Answer any **four**questions $2 \times 4 = 8$

   (a) Let X and Y be normed spaces. If X is finite dimensional, then show that every linear map from X to Y is continuous.

   (b) Give an example of a normal operator which is not self-adjoint.

   (c) Let X be a normed space. Show that $x_n \to x$ weakly in X does not imply $x_n \to x$ in X in general.

   (d) Let X and Y be normed spaces and $\mathrm{T} \in \mathrm{B}(X, Y)$. Show that if T is an isometry then $\|T\|$=1.

   (e) If $(T_n)$is a sequence of bounded linear-operators on a Hilbert-space H, and $T_n \to T$,show that$T_n^* \to T^*$.

   (f) Let $\mathrm{T} \in BL(H)$be a normal operator. Then show that Ker(T)=Ker(T^*).

2. (a)State and prove Hahn-Banach theorem for normed-spaces. 5

   (b) Define weak convergence of a sequence (x_n) in a normed space X .
   Show that weak-limit is unique. 3

3. (a) Let X be a vector space of all real- valued functions on $[0,1]$ that have continuous derivatives with the sup norm. Also let Y= $C[0,1]$ with sup norm.

Define $D: X \to Y$ by $Df = f'$

(i) Show that D is an unbounded linear operator.

(ii) Show that D has a closed graph.

(iii)Why does the conclusion in(ii) not contradict the closed graph theorem?

(b) Give an example to show that the completeness of the domain is an essential requirement in the Uniform Boundedness Principle. 5+3

4. (a) Let $P \in BL(\mathrm{H})$ be a nonzero projection on a Hilbert space H and $\|P\| = 1$. Then show that P is an orthogonal projection. 5

(b) Show that $Ran(T) = Ran(T^*)$ if $T \in BL(\mathrm{H})$ is a normal and H is a Hilbert space. 3

5. (a)State Reisz's lemma. Write an application of Reisz's lemma.

(b) Prove that $\mathrm{B}(R, Y)$ is not a Banach space if Y is not a Banach space. 3+5

6. (a) Prove that the adjoint operator has the same norm as the bounded operator itself.

(b) Consider the normed space $(C[-1,1], \|.\|_\infty), \|x\|_\infty = \sup\limits_{t \in [-1,1]} |x(t)|$. Show that the functional

$f(x) = \int_{-1}^{0} x(t)\,dt - \int_{0}^{1} x(t)\,dt$ is linear and bounded.

3+5

[Internal Assessment: 10 Marks]