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M.Sc 2nd Semester Examination, 2021

Applied Mathematics with Oceanology and Computer Programming

(General Topology)

Paper: MTM – 206

Full Marks:20 Time: 2 hours

The figures in the right-hand margin indicate marks

1. Answer any **two** questions $\mathbf{2 \times 2 = 4}$

   (a) Show that the order topology on a nonempty set is a Hausdorff space.

   (b) Show that subspace of a Hausdorff space is Hausdorff.

   (c) In the finite complement topology on **R,** to what point/points does the sequence (x_n) , where $x_n=n^2$ converge?

   (d) Determine closure of the interval A= (-2, $\sqrt{2}$) in the K-topology on **R**

2. Answer any **two** questions $\mathbf{2 \times 4 = 8}$

   (a) Define homeomorphism. Show that the subspace (0, 1) of **R** is not homeomorphic with unit disk in the plane.

   (b) Consider the product and box topology on $\mathbb{R}^N$. Under what topology the function $f: \mathbb{R} \to \mathbb{R}^{\mathbf{N}}$ defined by $f(t) = (t, \frac{t}{2}, .\frac{t}{3}, ...)$ is continuous?

   (c) Consider $f: X \to Y$ be a bijective continuous function. Show that if X is compact and Y is Hausdorff then f is a homeomorphism.

   (d) Define path connected space. Verify whether image of a path connected space under a continuous map is path connected.

3. Answer **any one** question: $1 \times 8 = 8$

(a) (i) Show $f : X \to Y$ is an open map if and only if $f(\text{int } A) \subseteq \text{int}(f(A))$ for each $A \subset X$, X and Y are topological spaces.

(ii) Show that the countable collection $B = \{[a,b) : a < b, a, b \in \mathbb{Q}\}$ is a basis that generates a topology different from the lower limit topology on $\mathbb{R}$.

(b) (i) Show that image of a compact space under a continuous map is compact.

(ii) Show that in the finite complement topology on $\mathbb{R}$, every subspace of **R** is compact.