M.sc 1st Semester Examination,2021

Applied Mathematics With Oceanology And Computer Programming

Paper: MTM – 106

(Graph Theory)

Full Marks:25 Time:1 hour

The figures in the right-hand margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
Illustrate the answers wherever necessary

1. Answer any ***two*** questions: 2x2=4
   (a) It is possible to draw a simple graph with 4 vertices and 7 edges? Justify.
   (b) If a graph G of order n is isomorphic its complement, show that n or $(n-1)$ must be a multiple of 4.
   (c) Define fundamental cut-set of a graph *G*.
   (d) If a tree has two centers, show that they must be adjacent.

2. Answer any ***two*** questions: 2x4=8
   (a) A simple graph with *n* vertices and *k* components cannot have more than $\frac{(n-k)(n-k+1)}{2}$ edges.
   (b) Define isomorphism of graphs. Give an example of two graphs that have the same number of vertices, edges, and degrees of vertices, but that are not isomorphic.
   (c) Prove that the chromatic polynomial of any cycle C_n of length *n* is $p_n(\lambda) = (\lambda-1)^n + (-1)^n(\lambda-1)$.
   (d) If *G* is connected planar graph with $n\ (\geq 3)$ vertices and *e* edges, then prove that $e \leq 3n-6$. Also, show that a simple connected planar graph with 6 vertices and 12 edges, each of the face is bounded by 3 edges.

3. Answer any ***one*** questions: 1x8=8
   (a) Define Eulerian graph. Give an example of a Eulerian graph which is not Hamiltonian. Prove that a connected graph G is Eulerian if and only if its vertices are all even degrees.
   (b) Define planar graph. Prove that in a connected planar graph of order n, size e and r regions $n - e + r = 2$.

[Internal Assessment: 05 Marks]