বিদ্যাসাগর বিশ্ববিদ্যালয়

VIDYASAGAR UNIVERSITY

Question Paper

B.Sc. Honours Examinations 2021

(Under CBCS Pattern)

Semester - III

Subject : STATISTICS

Paper : C 5-T

(Linear Algebra and Numerical Analysis)

Full Marks : 60

Time : 3 Hours

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.

(Theory)

Group - A

A. Answer any ***four*** of the following questions : 12×4=48

1. (i) Prove that the adjoint of a skew-symmetric determinant of order n is symmetric if n is odd and skew-symmetric if n is even. 6

   (ii) Find the basis and dimension of the vector space W of R^3 where

   $W = \{(x, y, z) \in R^3 \ : \ x + y + z = 0\}$ 6

2. (i) Evaluate $\int_1^2 \frac{dx}{2x-1}$ by Simpson's 1/3 rule by taking 4 subintervals. 6

(ii) If X_1, X_2, X_3 be three eigen vectors of a square matrix A over a field F corresponding to three distinct eigen values λ_1, λ_2, λ_3 respectively, then show that X_1, X_2, X_3 are linearly independent. 6

3. (i) For the given set of values for $y = f(x)$,

X	1	3	5	7	9
Y	8	12	21	36	62

form the backward difference table and hence find the values of $\Delta f(3)$, $\Delta^2 f(7)$ and $\Delta^3 f(9)$. 6

(ii) Express the matrix $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$ as the product of two matrices, each of rank 2. 6

4. (i) Find the interpolating polynomial $p_2(x)$ which interpolates $f(x) = \sin \pi x$ at $x = 0, \frac{1}{6}, \frac{1}{2}$ using Lagrange method. Also compute $p_2\left(\frac{\pi}{3}\right)$ 4+2

(ii) Show that the matrix $\begin{pmatrix} 1 & 2 & 2 \\ -2 & -1 & 2 \\ 2 & -2 & 1 \end{pmatrix}$ is orthogonal and hence solve the system of equations $x + 2y + 2z = 2$, $-2x - y + 2z = 1$, $2x - 2y + z = 7$. 3+3

5. (i) If A be a $m \times n$ matrix and P be a non-singular $n \times n$ matrix, then prove that the column rank of A is equal to the column rank of AP. 6

(ii) Examine whether $V = \{(a, b) : a, b \in R\}$ is a subspace for the following cases when vector addition and scalar multiplication are defined as : 3+3

(a) $(u, v) + (w, x) = (u + w, 0)$ and $c(u, v) = (cu, cv)$; $c \in R$

(b) $(u, v) + (w, x) = (u + w, v + x)$ and $n(u, v) = (u, nv)$; $n \in R$

6. (i) Show that the quadratic form $x^2 + 2y^2 + 3z^2 - 2xy + 4yz$ is indefinite. 4

(ii) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 4 & 4 \\ 3 & 3 & 7 \end{bmatrix}$ using elementary row operations.

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(iii) Determine the linear mapping $T : R^3 \to R^3$ which maps the basis vectors (1, 0, 0), (0, 1, 0), (0, 0, 1) of R^3 to the vectors (1, 1), (2, 3), (3, 2) respectively. Find T(1, 0, 0) and T (6, 0, 1). 4

Group - B

B. Answer any ***three*** of the following questions : 4×3=12

7. Show that a subset of a set of linear independent vectors is linearly independent.

8. Express $A = \begin{pmatrix} 4 & 5 & 1 \\ 3 & 7 & 2 \\ 1 & 6 & 8 \end{pmatrix}$ as the sum of a symmetric matrix and a skew symmetric matrix.

9. Calculate f (0.35) from the following table using Newton forward difference method :

X	0.3	0.5	0.6
f(x)	0.6179	0.6915	0.7257

10. Prove by Laplace method, $\begin{vmatrix} a & -b & -a & b \\ b & a & -b & -a \\ c & -d & c & -d \\ d & c & d & c \end{vmatrix} = 4\left(a^2+b^2\right)\left(c^2+d^2\right)$