



বিদ্যাসাগর বিশ্ববিদ্যালয়  
VIDYASAGAR UNIVERSITY  
Question Paper

**B.Sc. General Examinations 2022**  
(Under CBCS Pattern)  
**Semester - IV**  
**Subject : MATHEMATICS**  
**Paper : SEC 2 - T**

**Full Marks : 40**

**Time : 2 Hours**

*Candidates are required to give their answers in their own words as far as practicable.  
The figures in the margin indicate full marks.*

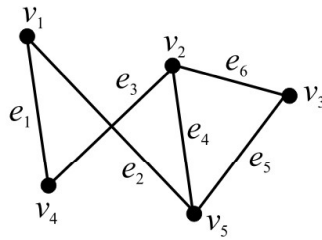
[ GRAPH THEORY ]

**Group - A**

Answer any **four** questions :

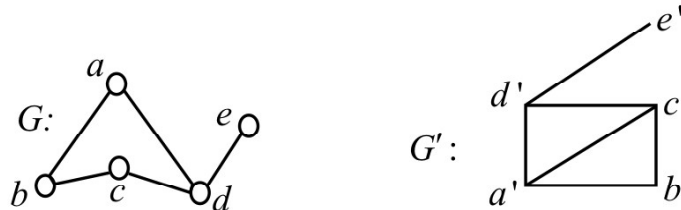
5×4=20

1. Define adjacency matrix of an undirected graph with  $n$  vertices. Find the adjacency matrix of the following graph. 2+3



P.T.O.

2. Define an Eulerian graph. Prove that a connected graph  $G$  is an Eulerian graph if it can be decomposed into circuits. 1+4
3. When two graphs are called isomorphic to each other? Check whether the following two graphs are isomorphic or not. 2+3

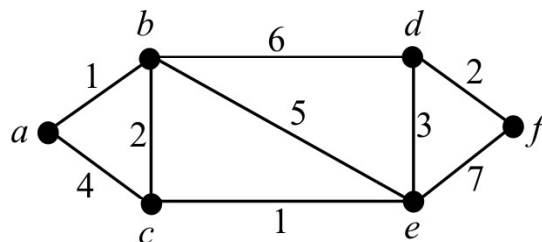


4. Define a Hamiltonian graph with an example. Draw a graph which is Eulerian but not Hamiltonian. What are the main differences between Eulerian circuit and Hamiltonian cycle? 2+1+2
5. Define a pseudo graphs and draw it. Prove that the number of vertices of odd degree in a graph is always even. 1+1+3
6. Define a connected graph. Prove that a graph  $G$  is disconnected if and only if its vertex set  $V$  can be partitioned into two non-empty, disjoint subsets  $V_1, V_2$  such that there exists no edge in  $G$  whose one end vertex is in  $V_1$  and other end vertex is in  $V_2$ . 1+4

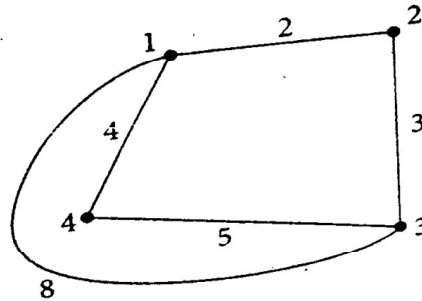
**Group - B**

Answer any **two** questions : 10×2=20

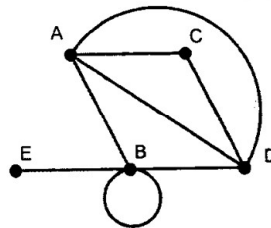
7. Define the length of a path in a weighted graph. Using Dijkstra's algorithm, find the shortest path between the vertices  $a$  and  $f$  on the following graph. 2+8



8. Using Floyd–Warshal’s algorithm find all pair of shortest paths from the following weighted graph :



9. Define graph and write the names of three important graphs. Prove that the degree of a graph is always even. Find the degree of all vertices of the following graph and verify the Euler formula  $v - e + f = 2$  for the same graph, where  $v$  the number of vertices,  $e$  is the number of edges and  $f$  is the number of faces (regions bounded by edges, including the outer, infinitely large region). 3+2+2+3



10. Let A, B, C, D, E be the five villages. We would like to connect these villages by a network of pipelines to supply water. The following table shows that the distances in units of 5 km between these five villages. Find a minimal Spanning tree connecting the five villages using Kruskal’s algorithm.

	A	B	C	D	E
A	-	2	4	3	5
B	2	-	7	4	6
C	4	7	-	10	8
D	3	4	10	-	9
E	5	6	8	9	-

OR

## [ INTEGRAL CALCULAS ]

1. Answer any **four** questions :

5×4=20

(a) Evaluate :  $\int \frac{x^2 dx}{(x-1)^3(x+1)}$

(b) If  $I_{m,n} = \int_0^1 x^m (1-x)^n dx$ , where  $m, n$  are positive integers, prove that

$$(m+n+1)I_{m,n} = nI_{m,n-1}$$
 and hence deduce the value of  $I_{m,n}$ .

(c) Show that the integral  $\iint e^{\frac{y-x}{y+x}} dx dy$  taken over the region enclosed by the triangle with vertices at  $(0, 0)$ ,  $(0, 1)$ ,  $(1, 0)$  is  $\frac{1}{4} \left( e - \frac{1}{e} \right)$ .(d) Show that the length of the arc of the parabola  $\frac{l}{r} = 1 + \cos \theta$  cut off by its latus ractum is  $l \left\{ \sqrt{2} + \log(1 + \sqrt{2}) \right\}$ .(e) Find the area of the segment of the parabola  $y = x^2 - 7x + 9$  cut off by the line  $y = 3 - 2x$ .(f) Find the volume generated by revolving x-axis, where the area is bounded by  $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ .2. Answer any **two** questions :

10×2=20

(a) Prove that the area included between the folium of Descartes  $x^3 + y^3 = 3axy$  and its asymptote  $x + y + a = 0$  is equal to the area of its loop.

P.T.O.

(b) (i) Obtain the intrinsic equation of the catenary  $y = a \cosh\left(\frac{x}{a}\right)$ , taking the vertex  $(0, a)$  as the fixed point. 5

(ii) Prove that  $\lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{n^2}{(n^2 + r^2)^{\frac{3}{2}}} = \frac{1}{\sqrt{2}}$ . 5

(c) If  $I_n = \int \frac{\sin(2n-1)x}{\sin x} dx$  and  $J_n = \int \frac{\sin^2 nx}{\sin^2 x} dx$ , show that

(i)  $n(I_{n+1} - I_n) = \sin 2nx$  ; (ii)  $J_{n+1} - J_n = I_{n+1}$  5+5

(d) (i) Prove that  $\iiint \frac{dx dy dz}{x^2 + y^2 + (z-2)^2} = \pi \left(2 - \frac{3}{2} \log 3\right)$  extended over the sphere

$$x^2 + y^2 + z^2 \leq 1. \quad 5$$

(ii) Find the perimeter of the cardioid  $r = a(1 - \cos \theta)$ . 5

OR

## [ MATHEMATICAL FINANCE ]

1. Answer any **four** questions : 5×4=20

- (a) What are the basic differences between simple and compound interest. Discuss the process of continuous compounding.
- (b) What is present and future value of a stream. Determine the relation between them.
- (c) A young couple has made a non-refundable deposit of first month's rent (equal to \$ 1000) on a 6-month apartment lease. The next day they find a different apartment that they like just as well, but its monthly rent is only \$ 900. They plan to be in the apartment only 6 months. Should they switch to the new apartment? What if they plan to stay 1 year? Assume an interest rate of 12%.
- (d) Discuss three components of an investor's required rate of return on an investment.
- (e) Briefly discuss about portfolio return. Find the mean and variance of portfolio return.
- (f) Discuss the relationship between Risk and Return.

2. Answer any **two** questions : 10×2=20

- (a) Write short note on (i) Business Risk, (ii) Financial risk, (iii) Liquidity risk, (iv) Exchange rate risk (v) Country risk.

- (b) Prove that the expected return  $\mu_i$  on any asset  $i$  satisfies  $\mu_i = r_f + \beta_i (\mu_M - r_f)$ ,

where  $\beta_i = \frac{\sigma_{iM}}{\sigma_M^2}$  and  $\sigma_{iM}$  is the covariance of the return on asset  $i$  and the market

portfolio  $r_m$  ;  $\sigma_M^2 = Var(r_M)$ .

- (c) Consider two projects whose cash flows are shown in the following table. Find IRR of the two projects and the NPVs at 5%. Show that the IRR and NPV figures yield different recommendation. Can you explain this ?

P.T.O.

	Year					
	0	1	2	3	4	5
Project - 1	-100	30	30	30	30	30
Project - 2	-150	42	42	42	42	42

- (d) (i) Your rate of return expectations for the stock of a company during the next year are : 6

Possible rate of return

Probability

-0.6

0.15

-0.3

0.1

-0.1

0.05

0.2

0.4

0.4

0.2

0.8

0.1

Compute expected return on this stock, the variance of this return and its SD.

- (ii) An 8% bond with 18 years of maturity has yield of 9%. What is the price of this bond? 4

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